Prophet inequalities with paid samples

Internship proposition to be supervised by Christoph Dürr, LIP6, Sorbonne University

Suppose you need to buy an airline ticket for your next vacations which start in n days. Every day you see a different price. Once you decided to buy the ticket the game is over. How should you decide when to buy? This is the topic of ongoing research on a problem called *prophet inequalities*.

Formally, n random variables $X_1, \ldots, X_n$ are drawn independently from an unknown distribution F. When you see the variable X_i, you can either decide to select it, and the game stops, or to wait for better days. The goal is to get the selected $E[X_i]$ as close as possible to $E[\max\{X_i\}]$ (for the maximization variant). The ratio between these values is called the *competitive ratio* and ideally should be as close as possible to 1. Since F is unknown to the algorithm, it is usually assumed that some number k of samples from F are initially given.

When $k = o(n)$, the ratio is roughly $1/e$ [Correa, Dütting, Fischer, Schewior, 2019], and when $k = O(n/\epsilon^6)$ a ratio of $0.745 - \epsilon$ is possible [Rubinstein, Wang, Weinberg, 2020], where 0.745 is the optimal ratio one can acheive when F is known. A recent work studied the case $k = \beta n$ for some constant β [Correa, Dütting, Fischer, Schewior, Ziliotto, 2021].

The goal of this internship is to define and study a model, where the algorithm can pay in order to get samples. There is clearly a trade-off to find between the performance guarantee of an algorithm and the cost it invests to learn F through paid samples. With a carefully chosen assumption on the domain of F, we could choose $E[X_i - k]$ as the profit of an algorithm choosing the i-th variable and paying for k samples, and study the competitive ratio of this reduced cost.

The roadmap would be to start reading the above mentioned papers, and then to work out small examples with simple distributions and small n, before trying to come up with more general results. If this works out nicely, it would lead to a publication in a good conference.